ANALYSIS OF A SERIES OF PRECISE DETERMINATION

OF SATELLITE POSITION IN A LONG ARC

By

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Introduction

Synchronous directional observations of artificial earth satellites, made by two or more earth stations, the position of which is known in one coordinate system, gives a possibility in determining the satellite's position in that same system of coordinates. In the near future there will enter another alternate method of complete determination of satellite position with the aid of observations of only one point, this will go into operation when the techniques of simultaneous directional determination of the satellite in the measurements of distances to it by new light measuring methods with the aid of lasers is achieved and developed. For the present, only the results of the first method will be used.

I. Purpose. Recently, the Smithsonian Astrophysical Observatory in the United States/SAO/ published a first catalog (1), containing data of synchronous photograph observations of several satellites achieved by a series of points of a known SAO network, with the aid of Baker-Nunn photographic cameras. For certain geodesic problems, there is significant importance attached to the observation orders which give a series of positions of the satellite in the significant portion of its geocentric arc, as a "chain" of its subsequent positions. In particular, in this work, the author has in mind the utilization of similar "chains" for the position determination of the mass center of the earth which plays an important role in the solution of basic problems of higher geodesy; that of the establishment of a single geocentric system of coordinates of earth stations. Principle determining of methods the mass-center of the earth utilizes such "chains" of satellite positions and these are presented by the author in work (2).

This method may be named by a conditional manner "a geometric one" and independent in the absence of implementation at the present time of a dynamic one. This method consists of a simultaneous derivation from a large quantity of non-synchronous observations as elements of observed satellites and also parameters of the gravitational pole of the earth and the geocentric coordinates of observation points. This brings us to a problem with somewhat large quantities of unknowns. This method was implemented in the SAO works which was established by the so-called "Standard Earth" (3) and produced quite valuable results, however, this was conducted using an enormous amount of observations.

The above mentioned "geometrical method" basically differs from it and can serve as an independent, control method using the results obtained previously, which can then serve as factors to increase the reliability of the desired values. However, for a complete utilization of the "geometrical method" it follows to have "chains" of satellite positions on sufficiently large geocentric arcs of their orbits and also for satellites with various values of declinations. This naturally requires special organizational observations, as in the distribution of the observation states, coordinates of which should be known in a single, geodesic/quasi-geocentric/coordinate system and also in relation to the parameters of orbits of such satellites, the observation of which is designated for such tasks. Up until the present time, no goal was set in this direction and there is insufficient data for a complete deduction of the desired quantities of this method.

The present work has a goal selected from the aforementioned SAO catalog of a sufficiently lengthy "chain" of observation which can develop a suitable method of processing the observed data and demonstrate that one similar "chains" gives as part of this required data which would complete the solution of this given problem.

2. <u>Initial</u> <u>data</u>. A series was selected from 66 satellite positions 1961 $\alpha\delta$, Midas 4, $i = 96^{\circ}$, $T = 166^{M}$, $e = 0.01$, $H = 3.5$ thousands km/, determined on October 29, 1963 on a geocentric arc approximately 57° with the aid of synchronous satellite observations with 4 SAO stations/Table 1/ containing Baker-Nunn instruments.

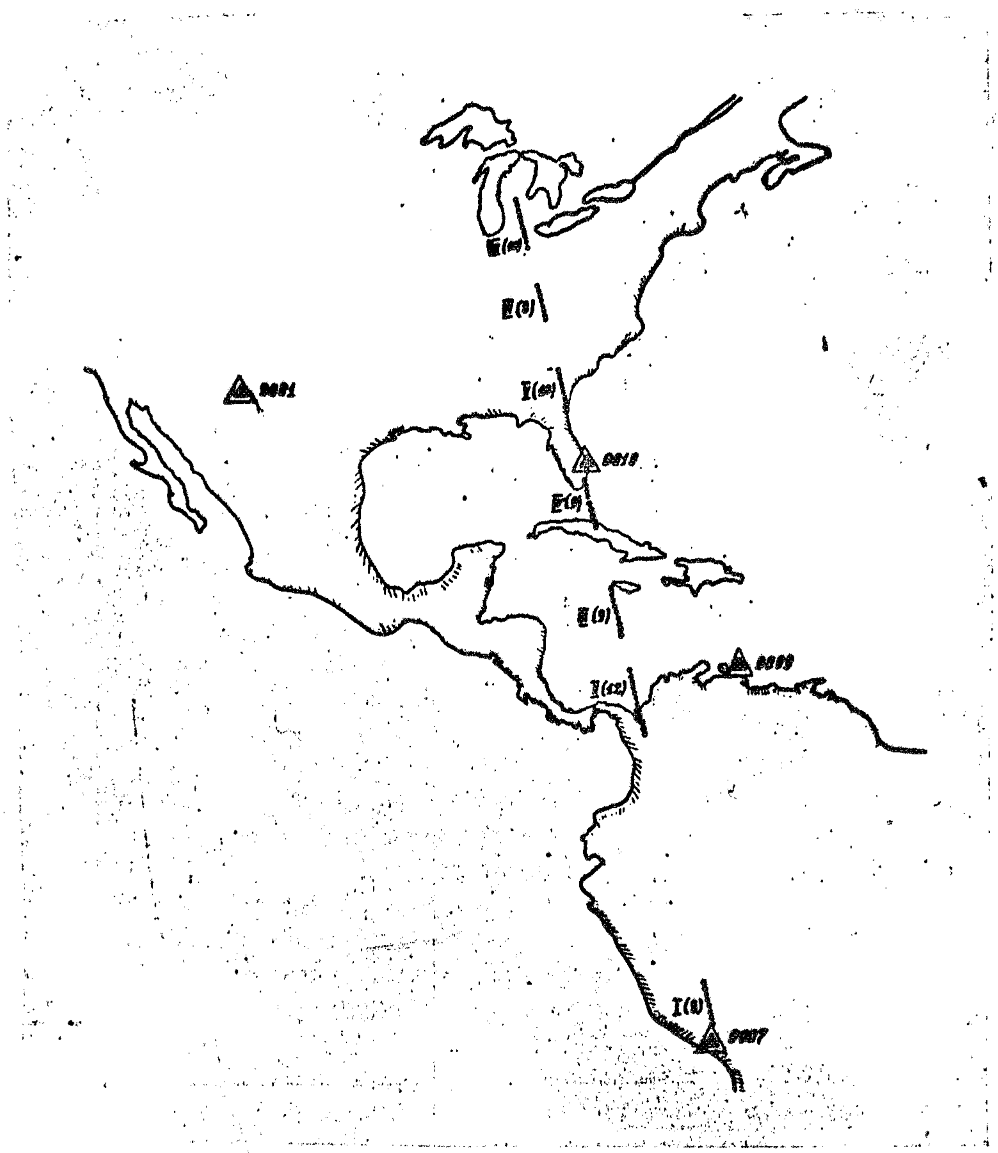

Figure 1

Table 1

Coordinates of Observation Stations

№	Название Name	B	$L_{зап}$	$H_м$	$X_м$	$y_м$	$Z_м$
9001	Organ Pass	$+32^{\circ}25'.4$	$106^{\circ}33'.1$	1651	-1535761	5167003	3401045.5
9010	Jupiter	+27 01.2	80 06.6	20	976284	5601398	2880247
9009	Curacao	+12 05.4	86 50.3	23	2251824	5816924	1327166
9007	Arequipa	-1628.1	71 29.6	2600	1942773	5804087	-1796964

The station distribution and the observed sub-satellite points are shown in the diagram of Figure 1.

The 28 satellite positions are determined with the aid of synchronous observations from two stations, the remaining 38 positions from three stations. (See table 2.) The observation commenced with two more southerly stations 9007 and 9009 approximately 1.5 hours before sunrise, when the satellite was in the southern hemisphere and well-lit by the rays of the sun. The observation was terminated after 26.5 m ins. when the most northerly station 9001 recorded a time of approximately 2:00 a.m. The entire series of satellite observation consists of 7 separate series, the duration of which is from 0.9 m to 1.5 m and containing 8 - 12 satellite positions. The altitude of the satellite over the surface of the earth changed in ranges of 3510-3540 km, the topocentric distance to the satellite from the station varied in ranges of 3520-5510 km.

3. Station Coordinates. In the examined problem, the station coordinates must be known in a single quasi-geocentric system which must be strictly

parallel to the geocentric system, that is, the possible declination of the system must be previously determined and calculated by its corresponding manner. With this goal in mind, one can take the North American NAD system with corrections for declination determined in work (4). However, the NAD system has limited distribution on the earth's surface which does not give observational possibilities in this system of satellites with long and various orbital arcs. This is why a system of coordinates was selected, in which the "chain" of SAO stations was determined, which girded the entire globe in the equatorial zone and was gradually supplemented with points in higher latitudes. The station coordinates in this system, as part of a so-called STANDARD EARTH, are presented in work (5). The S5 system was utilized for the analysis. Even though this system, which utilizes the methods of its determination, by concept should maximally approximate the geocentric system, however, for the general description of the methods the investigation of its declination was accomplished. This was done by a method shown in work (4). With this goal, results were utilized determining the absolute direction in an earth geocentric system with coordinates of 19 chords between 10 points (Area 1--the Atlantic, America, Pacific Ocean, which is presented in work 6). Into the number of these 10 points are included four stations shown in Table 1. From 38 corresponding adjustments the following quantities were obtained for angles α, β, γ, characterizing the desired declination of the S5 system.

$$\alpha = +0''.22 \pm 0''.76 \quad \beta = +0''.46 \pm 0''.55 \quad \gamma = -0''.21 \pm 0''.59 \quad /I/.$$

Since in the results were obtained very slight values of angles α, β, γ, and besides, in the limits of contingent errors of their determination, it was then possible to calculate this with sufficient accuracy with a possible slight declination of the system and to use these coordinates which interested us, in points of the indicated S5 system. These coordinates, designated as x_0, y_0, z_0, are given in an average earth geocentric system, which the Z_0 axis is directed by the average axis of earth rotation for the period 1900.0 to 1905.0 where plane X_0 Z_0 is inclined to the east at an angle $\Delta = 77^{\circ}55!94''$ from the average plane of the astronomical meridian of the U. S. Naval Observatory in Washington $\overline{\varphi_0} = 38^{\circ}55'14''$/, that is, the planes traversing through the direction of the perpendicular line of this observatory and parallel to the average axis of earth rotation (7) while axis Y_0 is directed 90° to the west x/ from the X_0 axis.

Since the utilized observations of Midas 4 were completed in October of 1963, then the coordinates of these observation points x, y, z were calculated in a multivenous earth geocentric system corresponding to the polar position in the indicated epoch. They are calculated by formula xx/

$$/x\ y\ z/ = /x_0\ y_0\ z_0/ \cdot \Phi, \qquad /2/$$

here and in further reference the matrix Φ are taken into consideration

$$\Phi = \begin{pmatrix} 1 & (\eta_0 \cos\Delta - \xi_0 \sin\Delta)\, tg\,\overline{\varphi_0}, & \xi_0 \\ -(\eta_0 \cos\Delta - \xi_0 \sin\Delta)\, tg\,\overline{\varphi_0} & 1 & \eta_0 \\ -\xi_0 & -\eta_0 & 1 \end{pmatrix} \qquad /3/$$

x/ For earth coordinates of axis, taken into consideration are the left system, that is the y axis is directed at 90° to the west from west axis. Therefore the coordinate signs of y indicated in the SAO works, where the right system is utilized they are changed in a reverse manner. What concerns the stellar systems, which is pointed out later, in this work, the right system is used.

xx/ Here and in subsequent indications are presented records of matrix products.

Where ξ_0-is the component of the polar transfer of the Z_oX_o plane to the south

η_0-is the component of the polar transfer perpendicular to ξ_0 to the west

Δ is the longitude of the basic observatory (latitude $\overline{\varphi}_0$) to the west from plane Z_o X_o.

Utilizing the polar rotation graph presented in work (7) for 29 October 1963 we obtain

$$\xi_0 = +0.55 \text{ м} = +8.64 \times 10^{-8} \underset{\text{рад.}}{\text{Rad.}}$$
$$\eta_0 = -0.45 \text{ м} = -7.07 \times 10^{-8} \underset{\text{рад.}}{\text{Rad.}}$$

Taking into consideration the values $\overline{\varphi}_0$ = 38°55'14" and Δ= 77°03'56", we obtain

$$\Phi = \begin{pmatrix} 1 & -5.71 \times 10^{-8} & +8.64 \times 10^{-8} \\ 5.71 \times 10^{-8} & 1 & -7.07 \times 10^{-8} \\ -8.64 \times 10^{-8} & 7.07 \times 10^{-8} & 1 \end{pmatrix} \qquad (5)$$

The results as ascertained by formula 2 we obtained coordinates of x, y, z of four observation stations presented in Table 1 in a multivenous earth geocentric system in the epoch of observations on 29 October 1963.

4. Preliminary processing of the observations. In catalog (1) for moments expressed in atomic time A1, coordinates of α and δ are given concerning directions of the "satellite observer" in a stellar coordinate system relating to the equator and to the ecliptical epoch of 1950.0. In these quantities, reductions for diurnal aberrations are made including the parallectic

refraction and the velocity of light. The indicated moments of quasi-synchronous satellite observations with several stations differ one from another no more than by four seconds. The first problem of this process was in the reduction of the indicated values α and δ to strict synchronous moments. In all cases it was possible to achieve this with the aid of linear interpolation.

By the obtained values, the directing cosigns were calculated in this manner

$$\bar{m}_0 = \cos\delta\cos\alpha, \quad \bar{n}_0 = \cos\delta\sin\alpha, \quad \bar{p}_0 = \sin\delta \qquad (6)$$

of the topocentrical directions to the satellite in a stellar geocentric system of coordinates for the 1950.0 epoch. In this system, the $\bar{Z}_0$ axis was directed toward pole and the $\bar{X}_0$ axis in a point to the Vernal Equinox of the given epoch, while the $\bar{Y}_0$ axis at 90° to the East. The synchronous moments AI and the obtained value I are presented in Table 2* in columns 2, 4, 5, 6.

A later problem consisted of the utilization of quantity 6, that is, to calculate the directing cosigns m, n, and p in topocentric directions to the satellite in relation to the earth geocentric coordinate systems for the observed epoch of 29 October 1963. In Tables 1 and 2 are shown the coordinate points of the observation stations. This is an equivalent, turn in accordance to the presession influence and nutation of the stellar system 1950.0 around the beginning coordinate to the epoch of 29 October 1963. Then a turn about the polar axis to the east at angle S equal to the stellar

* See end of article

Greenwich time in the moment of observation and finally to the change of direction of the y axis to the reverse**.

Conforming calculations for each of the 170 observations were completed by

$$/m\ n\ p/ = /\bar{m}_o\ \bar{n}_o\ \bar{p}_o/ \cdot P \cdot N \cdot S, \qquad (7)$$

where $/m\ n\ p/$ - is the line matrix of the desired quantities

$/\bar{m}_o \bar{n}_o \bar{p}_o/$ - is the line matrix of quantities 6

P - is the square matrix of precession

N - is the square matrix of nutation

S - is the square symmetrical matrix of stellar time.

The matrix of precession P is (supplement 2)

$$P = \begin{pmatrix} -\sin\xi_o \sin z + \cos\xi_o \cos z \cos J & \sin\xi_o \cos z + \cos\xi_o \sin z \cos J & \cos\xi_o \sin J \\ -\cos\xi_o \sin z - \sin\xi_o \cos z \cos J & \cos\xi_o \cos z - \sin\xi_o \sin z \cos J & -\sin\xi_o \sin J \\ -\cos z \sin J & -\sin z \sin J & \cos J \end{pmatrix} \qquad (8)$$

Angles z, ξ_o, J are determined from the following expression

$$\begin{aligned} z &= 23''.04952t + 1''.095 \times 10^{-4}t^2 + 0''.018 \times 10^{-6}t^3 \\ \xi_o &= 23''.04952t + 0''.302 \times 10^{-4}t^2 + 0''.018 \times 10^{-6}t^3 \\ J &= 20''.04259t - 0''.427 \times 10^{-4}t^2 - 0''.042 \times 10^{-6}t^3, \end{aligned} \qquad (9)$$

where t is expressed in tropical years from 1950.0. Expression (9) is obtained from basic formulas for the precession calculations taken from

** According to established agreement that in the earth's system the Y axis goes to the <u>West</u> from the X axis while in the stellar system in the <u>East</u>

Andoie by Newcomb's data (8, page 262). For the average observation moment of satellite Midas 4, for 29 October 1963 we obtained

$$z = 5'18''.6790 \quad \zeta_0 = 5'18''.6638 \quad J = 4'37''.0791 \tag{10}$$

With these values of the precession matrix, we obtain the following values

$$P = \begin{pmatrix} 0.99999432_{39} & 0.00308991_{88} & 0.00134331_{54} \\ -0.00308991_{88} & 0.99999522_{62} & -0.00000207_{53} \\ -0.00134331_{54} & -0.00000207_{54} & 0.99999909_{77} \end{pmatrix} \tag{11}$$

The nutation matrix N has form (appendix 3)

$$N = \begin{pmatrix} \cos\Delta\psi & \Delta\sin\Delta\psi\cos\varepsilon & \sin\Delta\psi\sin\varepsilon \\ -\sin\Delta\psi\cos\varepsilon_0 & \sin\varepsilon\sin\varepsilon_0+\cos\varepsilon\cos\varepsilon_0\cos\Delta\psi & -\cos\varepsilon\sin\varepsilon_0+\sin\varepsilon\cos\varepsilon_0\cos\Delta\psi \\ -\sin\Delta\psi\sin\varepsilon_0 & -\sin\varepsilon\cos\varepsilon_0+\cos\varepsilon\sin\varepsilon_0\cos\Delta\psi & \cos\varepsilon\cos\varepsilon_0+\sin\varepsilon\sin\varepsilon_0\cos\Delta\psi \end{pmatrix}, \tag{12}$$

where ε_0, declination of the average equated to the ecliptic

$\varepsilon = \varepsilon_0 + \Delta\varepsilon$ is the declination of the true equator to the ecliptic

$\Delta\varepsilon$ is a declination nutation (long period + short period)

$\Delta\psi$ is the nutation in the longitude.

For the average observation moment of 29 October 1963 $T_0 = 8^h.7$ from the Annual Astronomical Bulletins USSR for 1963 we obtain

$$\begin{aligned} \varepsilon_0 &= 23^\circ 26' 38''.366 \\ \varepsilon &= 23^\circ 26' 36''.245 \\ \Delta\psi &= -18''.092 \end{aligned} \tag{13}$$

With these values, the notation matrix obtains the following value

$$N = \begin{pmatrix} 0.99999999_{62} & -0.00008047_{2I} & -0.00003489_{58} \\ 0.00008047_{I8} & 0.99999999_{67} & -0.0000I028_{43} \\ 0.00003489_{66} & 0.0000I028_{I5} & 0.99999999_{28} \end{pmatrix} \quad (14)$$

Matrices P and N remain practically unchanged during the interval period of the observational processes, therefore the following single matrix was utilized equal to its product, with elements to 10^{-8} according to the accuracy of the station coordinates shown in formula 15

$$P.N = \begin{pmatrix} 0.99999462 & 0.00300945 & 0.00I30338 \\ -0.00300944 & 0.99999547 & -0.0000I225 \\ -0.00I3084I & 0.00000832 & 0.999999I4 \end{pmatrix} \quad (15)$$

The matrix of stellar time S has the following symmetrical form x/

$$S = \begin{pmatrix} \cos S & \sin S & 0 \\ \sin S & -\cos S & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (16)$$

Where S is the true stellar Greenwich time in the moment of each synchronous observation, which was selected from the Annual Astronomical Bulletin USSR for 1963 by the argument of universal time. For this, difference AI - UT1 = $+2^{s}.7562$, as utilized, obtained by the given data indicated in work (7, Tables 3 and 4).

x/ This is the kind of matrix S which forms the directing cosigns form of cosign $\cos\delta\cos\alpha$, $\cos\delta\sin\alpha$, $\sin\delta$ (in a right stellar system) considering $\cos\delta\cos t$, $\cos\delta\sin t$, $\sin\delta$, where $t = S - \alpha$ (in a left earth system).

The results of all calculations by formula 7, that is, the directing cosigns m, n, p of observed directions of the satellite are presented in Table 2 in columns 7, 8 and 9.

5. <u>The determination of right angle quasi-geocentric coordinates of satellites for all moments of synchronous observations</u>. The coordinates of four observation stations (Table 1) in the directing cosigns m, n, p (Table 2) for the satellite from these stations for the synchronous observation moments in one and the same coordinate system of a quasi-geocentric earth is strictly parallel to a geocentric system for the observation epoch of 29 October 1963. This gave the possibility of determining for all 66 synchronous moments, right angle coordinates of the satellites x_c, y_c, z_c in the same system. The calculations were performed for all instances (with observations from 2, as well as 3, stations) where the single system of formulas corresponding to the method indicated in the article (9). The utilized method gives the possibility of directly expressing three normal equations for more probable desired values of x_c, y_c, z_c, utilizing from the obtained observations the directing cosigns m, n, p and also station coordinates (no matter what their number) which take part in the given synchronous observations. Interpreting in geometrical terms the obtained results, it can be stated that the position of the satellite is determined by the point in space for which the sum of the squares of the length of the perpendiculars, dropped to the line of the observed directions (not intersecting in one point as a consequence of errors of these same observations taken as a station coordinate) will be minimal. After determining x_c, y_c, z_c it is easy to calculate length d of the indicated perpendiculars and the topocentric distance ρ to the satellite. Evidently,

relation $\frac{d}{\rho}$ presents angular corrections to the observed directions of the satellites, necessary for the intersection of these directions to the point with the obtained coordinates of the adjustment x_c, y_c, z_c. These quantities present interest as substantial characteristics of accuracy in a given system of synchronous observation (in this case with SAO points, Baker-Nunn instruments and Midas 4 satellite).

All of the coordinates of the satellite determined by these above described methods are shown in Table 2, Column 10. The calculated quantities for each observation length d equilibrating the perpendiculars, demonstrated a distance from 2 to 74 meters while the topocentric distances ρ from 3.5 to 5.5 thousand kilometers. Angular corrections $\frac{d}{\rho}$, are expressed in seconds of arc where in the limits of $0\overset{''}{.}I$ to $4\overset{''}{.}0$ while the average from the 170 values of these corrections has a quantity of $I\overset{''}{.}I$ from the mean arithmetical to the absolute declinations of separate values equal to $0\overset{''}{.}6$.

In column 2 of Table 2 are given quantities which equal the square root from the total squares of the coordinate errors x_c, y_c, z_c obtained from the deduction of these coordinates by the method of lesser squares. This quantity to a certain extent characterizes the accuracy of the satellite position derived from these figures.

6. <u>The perturbation determination of declination i and the longitude of the Ω node of the satellites</u>. The coordinates of the satellite x_c, y_c, z_c are deduced directly from the synchronous observations without any data of the elements of its orbit. However, in basic adjustments of examined problems (determination of coordinates ξ, η, ζ mass center of the earth two elements are considered: declination i and longitude of

node Ω, which are regarded as unknowns belonging to a certain series with quantities ξ , η , ζ . These elements to the extent of the entire length of the "chain" of the utilized observations (here 26.5 minutes) change somewhat following various perturbations, basically from the gravitational pole of the earth. So as not to increase the quantity of unknowns in these basic adjustments, it follows to take into account the changing of these perturbations between each separate observation in the whole "chain." This calculation is more expedient to achieve, utilizing not analytical formulas, but methods of computational integration. In the capacity of initial values of the coordinates and velocities of the satellite, it was necessary to make such an integration for the conduction of such an integration, certain values were taken calculated by the approximated elements of the satellite obtained directly by coordinates x_c, y_c, z_c of the satellite, shown in Table 2. This calculation was conducted by known formulas of theoretical astronomy (10) with units of tolerance, which indicate that the coordinates are geocentric. Computational integration was conducted on an IBM BESM-2 method, described in detail in work (2). The integration step equalled 0.5 minutes. In the gravitational field of the earth, all harmonics were taken into account (zonal and tessral) to the fourth order inclusively and besides that, the zonal harmonics of the fifth and sixth order to a total of 23 harmonics. The values of these harmonics were obtained from the data given in Standard Earth S5 (5, page 2). In this work were obtained corresponding perturbations of all elements in coordinates of the satellite extending throughout the period of the utilized observations, in that number of perturbations of element Ω and i, which were necessary for the problem which posed at that time. These perturbations in relation to the values of the elements which the satellite possessed in a certain average moment of all observations

$$S' = 11^h 11^m 15^s, \tag{17}$$

are presented in Table 3 for each of the 66 observations of the satellite positions. For unknown values of Ω' and i' in a given moment S' for later composition of basic adjustments of the problem, the following approximations of the quantities are taken

$$\begin{aligned} \Omega_0' &= 91°36'30'' \\ i_0' &= 95°51'24'', \end{aligned} \tag{18}$$

of corrections in which $\Delta\Omega$ and Δi are two or five unknowns (the other three ξ, η, ζ) entering into each of the basic adjustments of the examined problem.

Table 3

Perturbation of declination i and longitude of node Ω from the gravitational pole of earth.

№	$\delta\Omega$	δi	№	$\delta\Omega$	δi	№	$\delta\Omega$	δi
1	$-83^\circ x10^{-6}$	$+146^\circ x10^{-6}$	23	$+ 15^\circ x10^{-6}$	$+ 50^\circ x10^{-6}$	45	$+394^\circ x10^{-6}$	$+ 844^\circ x10^{-6}$
2	-81	+132	24	+ 17	+ 60	46	+406	+ 862
3	-79	+122	25	+ 20	+ 70	47	+418	+ 880
4	-77	+112	26	+ 23	+ 80	48	+430	+ 899
5	-75	+102	27	+ 27	+ 90	49	+574	+1101
6	-73	+ 92	28	+ 30	+101	50	+588	+1120
7	-71	+ 82	29	+ 33	+111	51	+603	+1139
8	-69	+ 68	30	+116	+331	52	+618	+1159
9	-27	-135	31	+122	+345	53	+633	+1178
10	-27	-130	32	+128	+359	54	+648	+1197
11	-26	-125	33	+135	+373	55	+664	+1216
12	-25	-120	34	+142	+388	56	+679	+1235
13	-24	-115	35	+148	+402	57	+814	+1393
14	-23	-109	36	+155	+417	58	+830	+1413
15	-23	-103	37	+162	+432	59	+848	+1433
16	-22	- 98	38	+168	+446	60	+867	+1453
17	-20	- 91	39	+328	+739	61	+885	+1473
18	-19	- 85	40	+339	+756	62	+904	+1493
19	-18	- 79	41	+349	+774	63	+923	+1513
20	-16	- 73	42	+360	+791	64	+942	+1533
21	+ 9	+ 32	43	+370	+808	65	+961	+1553
22	+12	+ 41	44	+381	+826	66	+981	+1573

7. <u>Basic equations for the determination of mass-center of earth. Results</u>. The theory of the examined method consists briefly of the following.x/ The plane equation of the satellite orbit in a geocentric system of coordinates (left) has the form

$$X \sin(\Omega - S)\sin i + Y \cos(\Omega - S)\sin i + Z\cos i = 0 \qquad (19)$$

Strict geocentric coordinates of satellite X, Y, Z are connected with observations of quasi-geocentric x, y, z relations (with both systems in parallel)

$$X = x + \xi, \quad Y = y + \eta, \quad Z = z + \zeta, \qquad (20)$$

where $-\xi$, $-\eta$, $-\zeta$ represent coordinates of the mass-center of earth in the utilized quasi-geocentric system. The value of Ω and i at a given moment of stellar Greenwich times are expressed as

$$\begin{aligned} \Omega &= \Omega_0' + \Delta\Omega + \delta\Omega \\ i &= i_0' + \Delta i + \delta i, \end{aligned} \qquad (21)$$

where Ω_0', i_0' are the approximated values for a certain moment S', $\Delta\Omega$, Δi of the desired corrections of these values, $\delta\Omega$, δi are the perturbations during the interval of time S - S', calculated from theoretical concepts. Substituting 20 and 21 in 19 and figuring $\Delta\Omega$, Δi, ξ, η, ζ as lesser quantities, we obtain the following form of basic equations of the examined problem with five unknowns.

$$a\Delta\Omega + b\Delta i + \alpha\xi + \beta\eta + \gamma\zeta + l = 0 \qquad (22)$$

The coefficients and the free member of this equation are calculated

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x/ More detailed explanation is given in works (2).

by the following formulas, where all quantities are known.

$$
\begin{aligned}
a &= (x\cos\tilde{\pi} - y\sin\tilde{\pi})\sin\rho \\
b &= (x\sin\tilde{\pi} + y\cos\tilde{\pi})\cos\rho - z\sin\rho \\
\alpha &= \sin\tilde{\pi}\sin\rho \\
\beta &= \cos\tilde{\pi}\sin\rho \\
\gamma &= \cos\rho \\
l &= (x\sin\tilde{\pi} + y\cos\tilde{\pi})\sin\rho + z\cos\rho \\
\tilde{\pi} &= \Omega_0' + \delta\Omega - S \\
\rho &= i_0' + \delta i
\end{aligned}
\qquad (23)
$$

Data presented in previous paragraphs of numeral results of processing the observations of satellite Midas 4 made in 29 October 1963 allow to compile 66 equations of form 22. Coefficients a, b, α, β, γ and the free member l of these equations are presented in Table 4. Unknowns $\Delta\Omega$ and Δi are expressed as seconds of arc, while the unknowns ξ, η, ζ in meters. Naturally, from one observation of this "chain" of the unknowns ξ, η, ζ which interest us, cannot be obtained separately, since the coefficients with these unknowns extending to all 66 equations do not change much. A slight change α and β occurs only after the turn of earth for all intervals of observations at an angle approximately $6\overset{\circ}{.}6$. For the determination of these unknowns it requires a joint processing of the "chains" of observations of other satellites with other declinations and with different positions of the body of the earth in relation to the plain of the satellite orbit. With obtained equations for Midas 4, only the analysis of approximate values of the total members $\alpha\xi + \beta\eta + \gamma\zeta$

can be made in equation 22. This united all of the terms in all 66 equations into one unknown

$$\delta = \bar{\alpha}\xi + \bar{\beta}\eta + \bar{\gamma}\zeta \qquad (24)$$

Table 4

Coefficients and free member of basic adjustments

№№	a	b	α	β	γ	ℓ
1	46.20152	12.45516	−0.95069	0.29287	−0.10204	+82.61613
2	46.26195	12.21557	−0.95086	0.29231	−0.10204	+49.01788
3	46.32216	11.97618	−0.95103	0.29176	−0.10204	+62.82535
4	46.37984	11.73624	−0.95120	0.29120	−0.10204	+38.91016
5	46.43728	11.49602	−0.95137	0.29065	−0.10204	− 6.873689
6	46.49456	11.25560	−0.95154	0.29009	−0.10204	+47.70689
7	46.54889	11.01465	−0.95171	0.28954	−0.10204	+50.49577
8	46.60257	10.77364	−0.95188	0.28898	−0.10204	+49.87231
9	47.19862	− 6.928236	−0.96318	0.24876	−0.10204	−24.44409
10	47.16148	− 7.173368	−0.96332	0.24820	−0.10204	+12.72076
11	47.12367	− 7.417824	−0.96346	0.24764	−0.10204	+46.29012
12	47.03330	− 7.662310	−0.96361	0.24708	−0.10204	+11.78672
13	47.04265	− 7.906406	−0.96375	0.24651	−0.10204	+27.30781
14	47.00012	− 8.150256	−0.96390	0.24595	−0.10204	+ 4.709017
15	46.95651	− 8.394241	−0.96404	0.24539	−0.10204	+37.34812
16	46.91172	− 8.637609	−0.96418	0.24483	−0.10204	+22.45760
17	46.86514	− 8.881213	−0.96433	0.24426	−0.10204	+20.59505
18	46.81833	− 9.124124	−0.96447	0.24370	−0.10204	+35.93893
19	46.76960	− 9.366835	−0.96461	0.24314	−0.10204	+27.52817
20	46.71978	− 9.609570	−0.96475	0.24258	−0.10204	+54.24979
21	45.96033	−12.73813	−0.96656	0.23525	−0.10204	− 7.457328
22	45.89366	−12.97654	−0.96670	0.23469	−0.10204	+ 7.725590
23	45.82545	−13.21474	−0.96684	0.23412	−0.10204	+ 2.992817
24	45.75621	−13.45239	−0.96697	0.23356	−0.10204	+27.30730
25	45.68547	−13.69009	−0.96711	0.23300	−0.10204	− 1.892297
26	45.61323	−13.92700	−0.96724	0.23243	−0.10204	+25.05252
27	45.54052	−14.16356	−0.96738	0.23187	−0.10204	+12.50063
28	45.46668	−14.40010	−0.96752	0.23130	−0.10204	+ 1.243742
29	45.39124	−14.63599	−0.96765	0.23074	−0.10204	−13.96351
30	43.83119	−18.81050	−0.97002	0.22057	−0.10205	−45.96847
31	43.73360	−19.03824	−0.97015	0.22000	−0.10205	−16.35080
32	43.63495	−19.26513	−0.97028	0.21944	−0.10205	− 9.30458
33	43.53465	−19.49169	−0.97040	0.21887	−0.10205	−13.29069
34	43.43328	−19.71165	−0.97053	0.21830	−0.10205	−19.27796
35	43.33099	−19.94334	−0.97066	0.21774	−0.10205	− 4.52662
36	43.22730	−20.16826	−0.97078	0.21717	−0.10205	−45.37000

(Table 4 continued)

№№	a	b	α	β	γ	ℓ
37	43.12249	-20.39248	-0.97091	0.21660	-0.10205	-37.80179
38	43.01681	-20.61609	-0.97104	0.21604	-0.10205	-50.46929
39	40.91970	-24.54685	-0.97325	0.20583	-0.10205	-15.10569
40	40.79318	-24.75955	-0.97337	0.20527	-0.10205	-35.33085
41	40.66487	-24.97137	-0.97349	0.20470	-0.10205	-14.75215
42	40.53569	-25.18265	-0.97361	0.20413	-0.10205	-26.36952
43	40.40577	-25.39301	-0.97373	0.20356	-0.10205	-33.96137
44	40.27439	-25.60292	-0.97385	0.20299	-0.10205	-14.81110
45	40.14201	-25.81213	-0.97397	0.20243	-0.10205	+10.47248
46	40.00878	-26.02051	-0.97408	0.20186	-0.10206	-25.39781
47	39.87435	-26.22844	-0.97420	0.20129	-0.10206	-36.77507
48	39.73902	-26.43554	-0.97432	0.20072	-0.10206	- 5.387311
49	38.18014	-28.66655	-0.97559	0.19447	-0.10206	-34.15118
50	38.03215	-28.86471	-0.97570	0.19390	-0.10206	- 1.363999
51	37.88323	-29.06220	-0.97581	0.19333	-0.10206	-28.54657
52	37.73352	-29.25903	-0.97592	0.19276	-0.10206	-51.26713
53	37.58254	-29.45494	-0.97604	0.19219	-0.10206	-11.80138
54	37.43091	-29.65019	-0.97615	0.19162	-0.10206	-27.63497
55	37.27813	-29.84504	-0.97626	0.19105	-0.10206	-28.46678
56	37.12422	-30.03844	-0.97636	0.19048	-0.10206	- 6.65941
57	35.85887	-31.55896	-0.97725	0.18593	-0.10206	-12.96405
58	35.69627	-31.74534	-0.97736	0.18536	-0.10206	- 3.301690
59	35.53312	-31.93103	-0.97746	0.18478	-0.10207	-94.87259
60	35.36868	-32.11564	-0.97757	0.18422	-0.10207	-10.22973
61	35.20327	-32.29906	-0.97768	0.18365	-0.10207	-13.93566
62	35.03706	-32.48228	-0.97779	0.18308	-0.10207	-39.08633
63	34.86989	-32.66434	-0.97789	0.18251	-0.10207	-18.30067
64	34.70181	-32.84558	-0.97800	0.18194	-0.10207	-14.78381
65	34.53291	-33.02585	-0.97810	0.18137	-0.10207	-15.53143
66	34.36319	-33.20555	-0.97821	0.18080	-0.10207	-47.98305

This changes the factual value of α, β, γ by their average values $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$ in all of the adjustments, which are obtained by values of

$$\bar{\alpha} = -0.96878, \bar{\beta} = 0.22338, \bar{\gamma} = -0.10205 \tag{25}$$

In this manner we obtain 66 equations as with 3 unknowns $\Delta\Omega$, Δi, σ, and this is solved by the method of lesser squares (taking into consideration all 66 equations in the form of 22)

$$\begin{aligned} \Delta\Omega &= +0''.12 \pm 1''.13 \\ \Delta i &= -1''.89 \pm 0''.36 \\ \sigma &= -36.1\text{m} \pm 53.1\text{m} \end{aligned} \tag{26}$$

with average error the unit weight $\varepsilon_0 = \pm\ 21.5$ m.

The small value of quantity σ demonstrates that the coordinate system S5, taken for the observation points, is rather close to the geocentric system, while the average error of this quantity that is trimember one (24) which permits the expectation that with the utilization of several similar "chains" of observations, which intersect in the center mass of earth, gives an opportunity to determine separately the required coordinates ξ, η, ζ with an accuracy of several tens of meter and perhaps less than that.

The obtained basic results of the coordinates of 66 satellite positions may be utilized in other ways as in the problem of the mass center of earth as well as for other purposes. Therefore, in this work these results are presented in proper detail.

Annex 1

Perturbation influence of poles in geocentric coordinates of points on the surface of earth.

In the auxilliary sphere (Figure 2) points Z_o and Z represent the direction of the northern portion of axis rotation of earth in the initial T_o epoch and in the given epoch T, while point M is the direction of the perpendicular line of the Greenwich observatory, Pulkovo and Washington, the latitude of which will be designated as $\bar{\varphi}_o$ and $\bar{\varphi}$. In the lines of equator AC_oB and ACB corresponding to the indicated epochs are distributed imprints of X_o, Y_o and X. In the earth geocentric coordinates of the axis, third axis of axis which are directed to point Z_o and Z. For the sake of generality we will assume that axis X_o and X are shifted to a certain angle Δ (to the east) from the meridians MN_o and MN. The polar shift from Z_o to Z is characterized by quantites s and θ. The angle ψ with point Z is auxilliary in further computation. The problem consists in the determination of reciprocal distribution of trihedrons of XYZ axis and $X_oY_oZ_o$.

Designating $\psi' = \psi + \Delta$ and $\Theta' = \Theta + \Delta$, we obtain

From the triangle

$$
\begin{array}{ll}
\cos(XX_0) = \sin\psi'\sin\Theta' + \cos\psi'\cos\Theta\cos s \ldots\ldots & XBX_0 \\
\cos(XY_0) = -\sin\psi'\cos\Theta' + \cos\psi'\sin\Theta'\cos s \ldots\ldots & XBY_0 \\
\cos(XZ_0) = -\sin s\cos\psi' \ldots\ldots & XZZ_0 \\
\cos(YX_0) = -\sin\Theta'\cos\psi' + \sin\psi'\cos\Theta'\cos s \ldots\ldots & YBX_0 \\
\cos(YY_0) = \cos\psi'\cos\Theta' + \sin\psi'\sin\Theta'\cos s \ldots\ldots & YBY_0 \\
\cos(YZ_0) = -\sin s\sin\psi' \ldots\ldots & YZZ_0 \\
\cos(ZX_0) = \sin s\cos\Theta' \ldots\ldots & ZZ_0X_0 \\
\cos(ZY_0) = \sin s\sin\Theta' \ldots\ldots & ZX_0Y_0 \\
\cos(ZZ_0) = \cos s \ldots\ldots &
\end{array}
\qquad /1/
$$

Furthermore from triangle ZZ_oM we obtain

$$\begin{aligned} \sin\bar{\varphi} &= \cos s \sin\bar{\varphi}_0 + \sin s \cos\bar{\varphi}_0 \cos\Theta, \\ \cos\bar{\varphi}\sin\psi &= \cos\bar{\varphi}_0 \sin\Theta, \\ \cos\bar{\varphi}\cos\psi &= -\sin s \sin\bar{\varphi}_0 + \cos s \cos\bar{\varphi}_0 \cos\Theta, \end{aligned} \tag{2}$$

which in concept solves the posed problem. By $\bar{\varphi}_0$, s, θ from (2) is determined the auxiliary angles ψ, after which in the equation (1) all quantities are known. However one can ignore the square of the small quantity of the shift of pole x and assume

$$\sin s = s, \cos s = 1, \tag{3}$$

after which we obtain

$$\begin{aligned} \sin\psi &= \sin\Theta + s\, tg\,\bar{\varphi}_0 \sin\Theta\cos\Theta \\ \cos\psi &= \cos\Theta - s\, tg\,\bar{\varphi}_0 \sin\Theta\sin\Theta \end{aligned} \tag{4}$$

substituting in equation (1) quantities (3) and (4), we find

$$\begin{matrix} \cos(XX_0)=1, & \cos(YX_0)=s\sin\Theta\, tg\,\bar{\varphi}_0, & \cos(ZX_0)=s\cos(\Theta+\Delta) \\ \cos(XY_0)=-s\sin\Theta\, tg\,\bar{\varphi}_0, & \cos(YY_0)=1, & \cos(ZY_0)=s\sin(\Theta+\Delta) \\ \cos(XZ_0)=-s\cos(\Theta+\Delta), & \cos(YZ_0)=-s\sin(\Theta+\Delta), & \cos(ZZ_0)=1 \end{matrix} \tag{5}$$

introducing for the position of the shifting pole the designations

$$\begin{aligned} &\xi_0 = s\cos(\theta+\Delta) \text{ into plane } Z_0X_0 \text{ to the south,} \\ &\eta_0 = s\sin(\theta+\Delta) \text{ perpendicularly } \xi_0 \text{ to the west,} \end{aligned} \tag{6}$$

$s \sin \theta = -\xi_0 \sin\Delta + \eta_0 \cos\Delta$ perpendicularly to meridian Z_0 M to the west, we obtain the latitudal matrix Φ presented in formula (3).

Annex 2

Precessions influence on coordinates and angular cosigns in a stellar coordinate system

Points $\bar{X}_o$, $\bar{Y}_o$, $\bar{Z}_o$ characterize in the auxiliary sphere (3) the direction of the coordinate axis of the stellar geocentric coordinate systems for the initial epoch T_o. Axis $\bar{Z}_o$ is directed toward the pole, axis $\bar{X}_o$ to the point of the vernal equinox γ_0 of this epoch, while axis $\bar{Y}_o$ at 90^o to the east from the axis $\bar{X}_o$. A change position of the axis after precession for certain epochs represents points $\bar{X}_1$, $\bar{Y}_1$, $\bar{Z}_1$ where axis $\bar{Z}_1$ and $\bar{X}_1$ are directed into corresponding and first point of γ_1, vernal equinox. The entire axis system is the average for epoch in absence of the true one which is obtained if notation is taken into account. The reciprocal distribution of the axis of two coordinates is characterized by the following value of 9, are all functions of 3 angle z, ζ_0, J, which are found in the figure.

From the triangle

$\cos(\bar{X}_1\bar{X}_0) = -\sin Z \sin\zeta_0 + \cos Z \cos\zeta_0 \cos J$	$X_1\ Q\ X_0$
$\cos(\bar{X}_1\bar{Y}_0) = -\sin Z \cos\zeta_0 - \cos Z \sin\zeta_0 \cos J$	$X_1\ Q\ Y_0$
$\cos(\bar{X}_1\bar{Z}_0) = -\cos Z \sin J$	$X_1\ Z_1\ Z_0$
$\cos(\bar{Y}_1\bar{X}_0) = \cos Z \sin\zeta_0 + \sin Z \cos\zeta_0 \cos J$	$Y_1\ Q\ X_0$
$\cos(\bar{Y}_1\bar{Y}_0) = \cos Z \cos\zeta_0 - \sin Z \sin\zeta_0 \cos J$	$Y_1\ Q\ Y_0$
$\cos(\bar{Y}_1\bar{Z}_0) = -\sin Z \sin J$	$Y_1\ Z_1\ Z_0$
$\cos(\bar{Z}_1\bar{X}_0) = \cos\zeta_0 \sin J$	$Z_1\ Z_0\ X_0$
$\cos(\bar{Z}_1\bar{Y}_0) = -\sin\zeta_0 \sin J$	$Z_1\ Z_0\ Y_0$
$\cos(\bar{Z}_1\bar{Z}_0) = \cos J$	—

Utilizing these values, we obtained a precession matrix P corrected in formula 8.

Annex 3

Nutation influence on coordinates and angular cosign in a stellar coordinate system.

In Figure 4 the given epoch T, point E is the ecliptical pole and AB ecliptic. $\bar{X}_1 \bar{Y}_1 \bar{Z}_1$ are the average stellar system of coordinates, whereby axis $\bar{Z}_1$ is directed to the average pole, $\bar{X}_1$ to the average point of the vernal equinox for the first point of γ_1, $\bar{Y}_1$ to 90° to the east. Arc E $\bar{Z}_1$ = $\bar{\varepsilon}$ is the declination of the average equator to the ecliptic. $\bar{X}$ $\bar{Y}$ $\bar{Z}$ is the corresponding multivenous stellar system of coordinates, the position of which is completely determined by the element of nutation

$\Delta\psi$ is the nutation in the longitude

$\Delta\varepsilon = \varepsilon - \bar{\varepsilon}$ is the declination nutation ,

and ε = the arc of E $\bar{Z}$ of the declination of multivenous equator, while the axis $\bar{X}$ directed to the true point of the vernal equinox γ.

Introducing into the triangular $\bar{Z}$ E $\bar{Z}_1$, the auxiliary quantities K and f, we determined then by the formulas

$$
\begin{aligned}
\cos K &= \cos\varepsilon \cos\bar{\varepsilon} + \sin\varepsilon \sin\bar{\varepsilon} \cos\Delta\psi, \\
\sin K \sin f &= \sin\bar{\varepsilon} \sin\Delta\psi, \\
\sin K \cos f &= \cos\bar{\varepsilon} \sin\varepsilon - \sin\bar{\varepsilon} \cos\varepsilon \cos\Delta\psi;
\end{aligned}
\tag{1}
$$

After which the cosign of the angles between the axis are easily determined in the indicated system of coordinates.

From the triangle

$$
\begin{array}{lll}
\cos(\overline{X}\overline{X}_1) = \cos\Delta\psi \ \ldots\ldots\ldots & X\,E\,X_1 & \\
\cos(\overline{X}\overline{Y}_1) = -\sin\Delta\psi\cos\bar{\varepsilon} \ \ldots\ldots & X\,X_1Y_1 & \\
\cos(\overline{X}\overline{Z}_1) = -\sin K\,\sin f \ \ldots\ldots & X\,X_1Z_1 & (2)\\
\cos(\overline{Y}\overline{X}_1) = \sin\Delta\psi\cos\varepsilon \ \ldots\ldots & Y\,X\,X_1 & \\
\cos(\overline{Y}\overline{Y}_1) = \sin\varepsilon\sin\bar{\varepsilon} + \cos\varepsilon\cos\bar{\varepsilon}\cos\Delta\psi \ldots & Y\,E\,Y_1 & \\
\cos(\overline{Y}\overline{Z}_1) = -\sin K\cos f \ \ldots\ldots\ldots & Y\,Z\,Z_1 & \\
\cos(\overline{Z}\overline{X}_1) = \sin\varepsilon\sin\Delta\psi \ \ldots\ldots\ldots & Z\,E\,X_1 & \\
\cos(\overline{Z}\overline{Y}_1) = -\cos\varepsilon\sin\bar{\varepsilon} + \sin\varepsilon\cos\bar{\varepsilon}\cos\Delta\psi \ldots & Z\,E\,Y_1 & \\
\cos(\overline{Z}\overline{Z}_1) = \cos K \ \ldots\ldots\ldots & \text{—} &
\end{array}
$$

Substituting the value (1) the matrix nutation of matrix N in the form presented in Formula (12).

If we ignore the squares of smaller quantities $\Delta\psi$ and $\Delta\varepsilon = \varepsilon - \bar{\varepsilon}$, then the nutation matrix with sufficient for a series of problems of approximation we can present this in the following form

$$
N = \begin{pmatrix} 1 & \Delta\psi\cos\varepsilon & \Delta\psi\sin\varepsilon \\ -\Delta\psi\cos\varepsilon & 1 & \Delta\varepsilon \\ -\Delta\psi\sin\varepsilon & -\Delta\varepsilon & 1 \end{pmatrix}, \qquad (3)
$$

Where $\Delta\psi \cos \varepsilon = \Delta\alpha$ is the nutation in a direct ascendant

$\Delta\psi \sin \varepsilon = \Delta\delta$ is nutation in the declinations.

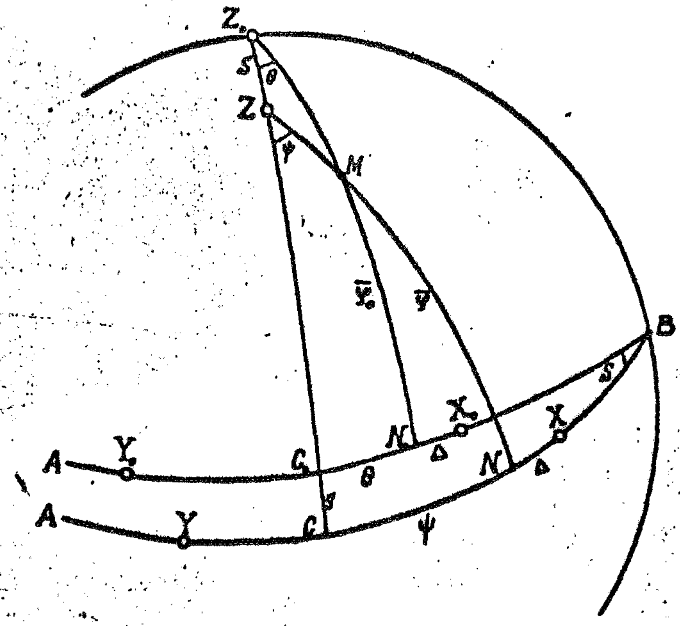

Рис. 2
Figure 2

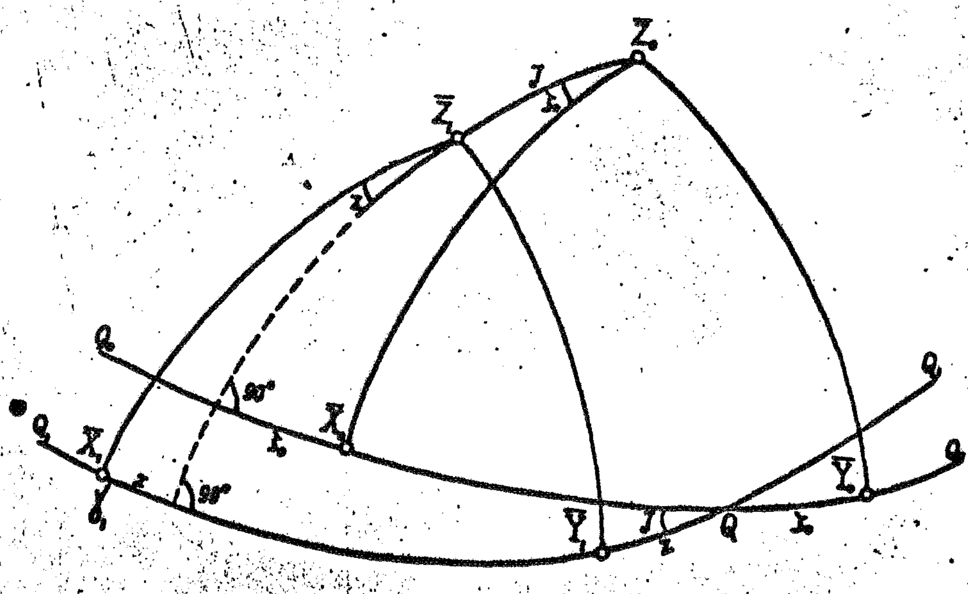

Рис. 3
Figure 3

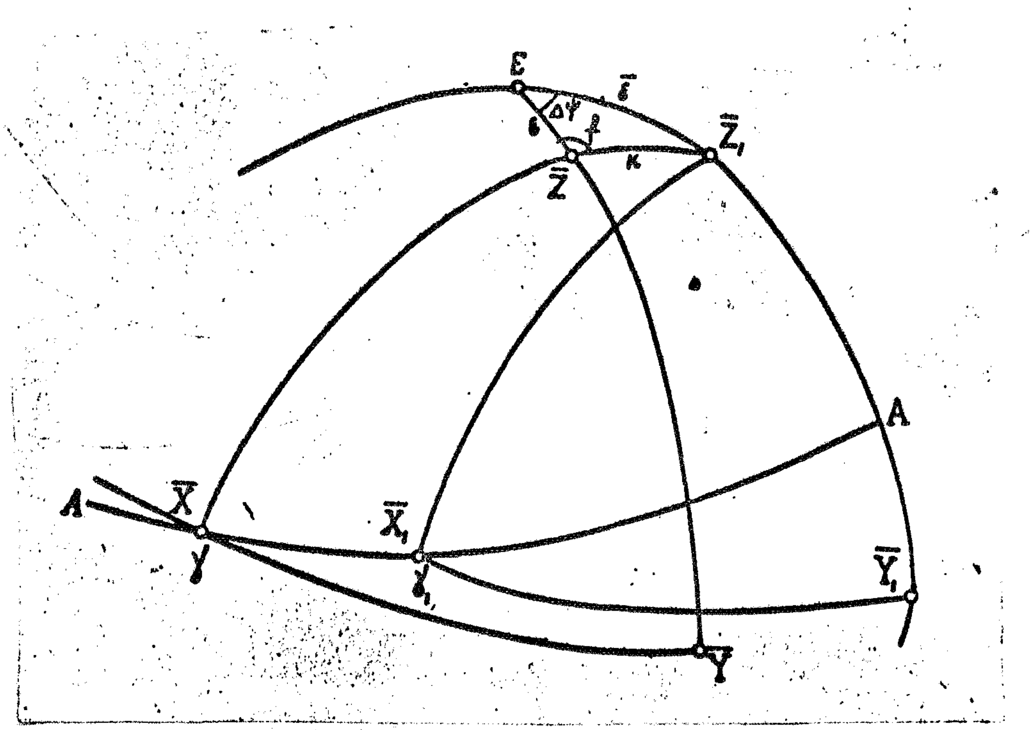

Figure 4

Initial results from observations of satellite Midas 4, 29 October 1963 Table 2

№№	Время 1 At Sгр.	2 Пункты наблюд.	3 Косинусы направления " наблюдатель – спутник "						6 Коорд. Спутника x_c y_c z_c (м)	$\pm\varepsilon$ (м)
			4 Звездная система 1950.0			5 Земная система 1963 X				
			$\bar{m}_0$	$\bar{n}_0$	$\bar{p}_0$	m	n	p		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		7	-.05693447	.97518744	-.21393437	.31820190	.92354898	-.21402062	+ 3 070 791.3	
1	$8^h 30^m 10^s.6250$	9	.01716028	.65438245	-.75596901	.15944718	.63490965	-.75595392	+ 9 078 029.3	10.5
	10 57 56.7238	11	-	-	-	-	-	-	- 2 555 659.5	
		7	-.05456271	.97828320	-.19996222	.31623217	.92735055	-.20004542	+ 3 064 387.5	
2	8 30 18.6250	9	.01890033	.66055670	-.75053823	.15907394	.64141545	-.75052094	+ 9 093 267.6	25.6
	10 58 04.7457	11	-	-	-	-	-	-	- 2 506 494.6	
		7	-.05216866	.98116746	-.18598078	.31417972	.93095244	-.18606090	+ 3 057 913.4	
3	8 30 26.6250	9	.02066513	.66674247	-.74500163	.15867258	.64793877	-.74498212	+ 9 108 470.2	47.4
	10 58 12.7676	11	-	-	-	-	-	-	- 2 457 376.5	
		7	-.04977055	.98384171	-.17198369	.31206286	.93435105	-.17206071	+ 3 051 294.4	
4	8 30 34.6250	9	.02244576	.67290350	-.73938966	.15824185	.65444222	-.73936790	+ 9 123 159.5	37.6
	10 58 20.7895	11	.08890724	.97232028	.21607585	.17587923	.96038152	.21618008	- 2 408 140.7	
		7	-.04737091	.98631164	-.15794097	.30988594	.93755106	-.15801489	+ 3 044 658.9	
5	8 30 42.6250	9	.02423558	.67907961	-.73366444	.15779893	.66096235	-.73364042	+ 9 137 792.4	19.1
	10 58 28.8114	11	.09022147	.96991979	.22608780	.17342186	.95852034	.22619376	- 2 358 846.2	
		7	-.04495251	.98856988	-.14390571	.30763067	.94054991	-.14397652	+ 3 037 898.8	
6	8 30 50.6250	9	.02607514	.68527668	-.72781588	.15730630	.66751563	-.72778953	+ 9 152 418.9	29.6
	10 58 36.8332	11	.09153504	.96743624	.23598401	.17094501	.95657645	.23609171	- 2 309 519.6	
		7	-.04252136	.99062005	-.12986092	.30530417	.94334931	-.12992858	+ 3 030 983.7	
7	8 30 58.6250	9	.02792047	.69143354	-.72190034	.15678985	.67403133	-.72187166	+ 9 166 444.0	29.1
	10 58 44.8552	11	.09282264	.96486560	.24580142	.16847214	.95453916	.24591084	- 2 260 079.8	
		7	-.04010567	.99245868	-.11583312	.30293304	.94593832	-.11589765	+ 3 024 015.2	
8	8 31 06.6250	9	.02978523	.69759481	-.71587312	.15624844	.68055605	-.71584208	+ 9 180 334.5	20.1
	10 58 52.8771	11	-	-	-	-	-	-	- 2 210 628.4	
		7	-	-	-	-	-	-	+ 2 306 111.6	
9	8 40 42.6200	9	.21135208	.97708298	.02528141	.01468800	.99956574	.02554595	+ 9 512 023.4	27.0
	11 08 30.4486	10	-.09466339	.93821567	-.33285164	.30357440	.89272753	-.33298670	+ 1 421 594.7	
		7	-	-	-	-	-	-	+ 2 293 608.6	
10	8 40 50.6200	9	.21399670	.97605882	.03891785	.01131499	.99916787	.03918585	+ 9 507 206.3	15.9
	11 08 38.4705	10	-.09318660	.94173571	-.32318743	.30241960	.89666392	-.32332061	+ 1 471 889.1	

1. Time; 2. Observation points; 3. Cosigns direction (satellite observer);
4. Earth system 1950.0; 5. Earth system 1963; 6. Satellite coordinates

(continuation)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		7	-	-	-	-	-	-	+ 2 281 095.3	
11	8 40 58.6200	9	.21660972	.97484389	.05253208	.00793036	.99857343	.05280350	+ 9 502 237.8	11.4
	11 08 46.4924	10	-.09168960	.94517904	-.31341602	.30122351	.90052900	-.31354729	+ 1 522 045.2	
		7	.10263978	.73298234	.67245964	.06503132	.73715750	.67258438	+ 2 268 521.6	
12	8 41 06.6200	9	.21920816	.97343208	.06616472	.00451660	.99778023	.06643954	+ 9 496 728.1	38.5
	11 08 54.5143	10	-.09019222	.94854332	-.30353079	.30000511	.90431601	-.30366015	+ 1 572 214.4	
		7	.10383569	.72719248	.67853448	.06214864	.73181761	.67866084	+ 2 255 899.6	
13	8 41 14.6200	9	-	-	-	-	-	-	+ 9 491 161.4	10.2
	11 09 02.5362	10	-.08864743	.95133030	-.29353144	.29871905	.90803712	-.29365884	+ 1 622 298.3	
		7	.10501447	.72138790	.68452279	.05928747	.72645601	.68465076	+ 2 243 232.0	
14	8 41 22.6200	9	.22429847	.97003385	.09340520	-.00233241	.99559900	.09368670	+ 9 485 194.1	40.5
	11 09 10.5581	10	-.08710375	.95503221	-.28341917	.29741100	.91167382	-.28354459	+ 1 672 335.8	
		7	.10617444	.71557272	.69042209	.05644747	.72107704	.69055165	+ 2 230 459.5	
15	8 41 30.6200	9	.22682087	.96803920	.10701588	-.00579853	.99420970	.10730069	+ 9 479 013.6	6.9
	11 09 18.5800	10	-.08550457	.95815832	-.27316955	.29602804	.91524771	-.27329293	+ 1 722 395.3	
		7	.10731847	.70976531	.69621546	.05363089	.71569909	.69634658	+ 2 217 693.2	
16	8 41 38.6200	9	-	-	-	-	-	-	+ 9 472 568.6	30.8
	11 09 26.6019	10	-.08390950	.96119078	-.26281455	.29462437	.91872803	-.26293588	+ 1 772 332.9	
		7	.10843989	.70393509	.70193744	.05083753	.71029088	.70207009	+ 2 204 827.5	
17	8 41 46.6200	9	-	-	-	-	-	-	+ 9 465 757.2	26.2
	11 09 34.6238	10	-.08227167	.96413747	-.25232974	.29315624	.92213281	-.25244898	+ 1 822 318.1	
		7	.10955987	.69813391	.70753493	.04805777	.70490771	.70766912	+ 2 191 950.1	
18	8 41 54.6200	9	-	-	-	-	-	-	+ 9 458 888.3	11.2
	11 09 42.6533	10	-.08062199	.96698802	-.24173180	.29165111	.92544515	-.24184892	+ 1 872 158.8	
		7	.11065870	.69231815	.71305697	.04530236	.69950264	.71319266	+ 2 179 022.3	
19	8 42 02.6200	9	-	-	-	-	-	-	+ 9 451 614.7	30.2
	11 09 50.6676	10	-.07895402	.96974000	-.23102079	.29010387	.92866356	-.23113577	+ 1 921 961.0	
		7	.11174328	.68650014	.71849216	.04256626	.69408932	.71862934	+ 2 166 008.5	
20	8 42 10.6200	9	-	-	-	-	-	-	+ 9 444 117.2	7.0
	11 09 58.6896	10	-.07724366	.97239627	-.22017925	.28849025	.93179654	-.22029204	+ 1 971 764.7	
		1	-.46625821	.87312702	-.14231121	.64063006	.75442933	-.14293183	+ 1 993 165.2	
21	8 43 54.6200	9	.26658363	.90058591	.34333392	-.06909788	.93654443	.34367139	+ 9 322 828.4	43.7
	11 11 42.9742	10	-.05298809	.99622012	-.06883131	.26295767	.96234314	-.06891278	+ 2 613 713.6	
		1	-.46678927	.87415590	-.13401208	.64093713	.75569401	-.13463341	+ 1 979 574.0	
22	8 44 02.6200	9	.26843885	.89522979	.35567433	-.07257445	.93165809	.35601428	+ 9 311 742.9	12.9
	11 11 50.9961	10	-.05095184	.99710459	-.05644782	.26060898	.96378820	-.05652665	+ 2 662 232.3	
		1	-.46732095	.87511358	-.12564779	.64122860	.75688957	-.12626984	+ 1 965 945.4	
23	8 44 10.6200	9	.27026289	.88972274	.36790137	-.07604684	.92661395	.36824376	+ 9 300 331.3	18.3
	11 11 59.0180	10	-.04890055	.99783648	.04394429	.25821194	.96508486	-.04402046	+ 2 711 507.2	
		1	-.46783048	.87600814	-.11723649	.64148402	.75802866	-.11785922	+ 1 952 263.3	
24	8 44 18.6200	9	.27203844	.88407287	.38001346	-.07949616	.92141627	.38035823	+ 9 288 699.6	68.9
	11 12 07.0399	10	-.04681397	.99841137	-.03135596	.25574606	.96623296	-.03142941	+ 2 760 268.8	

(continuation)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		1	-.46835138	.87682710	-.10872542	.64173374	.75909198	-.10934885	+ 1 938 573.1	
25	8 44 26.6200	9	.27378295	.87827824	.39200794	-.08293959	.91606689	.39235506	+ 9 276 742.2	34.7
	11 12 15.0618	10	-.04471836	.99882537	-.01865901	.25323651	.96722308	-.01872974	+ 2 809 043.2	
		1	-.46884838	.87758012	-.10017152	.64194531	.76009636	-.10079562	+ 1 924 787.4	
26	8 44 34.6200	9	-	-	-	-	-	-	+ 9 264 481.0	74.9
	11 12 23.0337	10	-.04259296	.99907503	-.00591159	.25066243	.96805609	-.00597955	+ 2 857 651.8	
		1	-.46932283	.87826595	-.09156964	.64211991	.76104023	-.09219437	+ 1 911 038.3	
27	8 44 42.6200	9	.27714594	.86628227	.41562622	-.08976991	.90493303	.41597786	+ 9 252 097.6	46.0
	11 12 31.1056	10	-.04045765	.99915711	.00694776	.24804295	.96872460	.00688258	+ 2 906 193.6	
		1	-.46979529	.87887657	-.08287561	.64227616	.76191136	-.08350098	+ 1 897 248.3	
28	8 44 50.6200	9	.27876798	.86009462	.42723022	-.09315693	.89916276	.42758405	+ 9 239 475.1	22.5
	11 12 39.1275	10	-.03829438	.99906814	.01991022	.24535998	.96922885	.01984786	+ 2 954 729.3	
		1	-.47025562	.87941070	-.07413827	.64240393	.76271064	-.07476425	+ 1 883 415.7	
29	8 44 58.6200	9	.28034352	.85377832	.43871414	-.09651863	.89325339	.43907010	+ 9 226 512.3	27.3
	11 12 47.1494	10	-.03611152	.99880495	.03293394	.24262126	.96956393	.03287443	+ 3 003 131.6	
		1	-.47563815	.87468662	.09322916	.63891991	.76368012	.09259605	+ 1 629 061.0	
30	8 47 22.6200	9	.30216354	.72332770	.62088182	-.15277745	.76856061	.62126777	+ 8 949 839.1	33.1
	11 15 11.5437	10	-	-	-	-	-	-	+ 3 859 693.5	
		1	-.47573171	.87353514	.10303255	.63835219	.76290292	.10239932	+ 1 614 662.0	
31	8 47 30.6200	9	.30303357	.71540449	.62957690	-.15562572	.76087180	.62996408	+ 8 932 121.8	25.7
	11 15 19.5656	10	.00857865	.95710153	.28962572	.18017146	.94002958	.28962498	+ 3 906 420.9	
		1	-.47581106	.87227551	.11286833	.63775051	.76202205	.11223501	+ 1 600 273.9	
32	8 47 38.6200	9	.30386440	.70745262	.63810440	-.15843246	.75314419	.63849275	+ 8 914 166.1	63.2
	11 15 27.5875	10	.01105210	.95284635	.30325186	.17638080	.93644359	.30325438	+ 3 952 974.2	
		1	-.47587093	.87090346	.12277626	.63710907	.76103427	.12214286	+ 1 585 849.8	
33	8 47 46.6200	9	.30467296	.69943356	.64650373	-.16122170	.74534335	.64689324	+ 8 895 859.8	50.3
	11 15 35.6094	10	.01351508	.94838451	.31683460	.17256418	.93264879	.31684040	+ 3 999 462.2	
		1	-.47589846	.86943229	.13269570	.63641798	.75995508	.13206228	+ 1 571 412.7	
34	8 47 54.6200	9	.30543853	.69138043	.65475217	-.16396656	.73749774	.65514277	+ 8 877 319.5	27.1
	11 15 43.6313	10	.01598346	.94370161	.33041158	.16870367	.92863409	.33042064	+ 4 045 830.7	
		1	-.47589929	.86784956	.14267797	.63568056	.75877109	.14204456	+ 1 556 941.7	
35	8 48 02.6200	9	.30617588	.68329526	.66284532	-.16668105	.72961232	.66323698	+ 8 858 581.8	27.3
	11 15 51.6532	10	.01846934	.93881060	.34393827	.16479022	.92441449	.34395064	+ 4 092 134.2	
		1	-.47588979	.86614739	.15270105	.63491153	.75747131	.15206766	+ 1 424 499.7	
36	8 48 10.6200	9	.30688007	.67517827	.67078978	-.16936024	.72168638	.67118245	+ 8 839 532.6	44.6
	11 15 59.6751	10	.02093571	.93371577	.35740252	.16086125	.91998691	.35741816	+ 4 138 290.5	
		1	-.47583726	.86434669	.16273815	.63408298	.75608253	.16210484	+ 1 527 997.8	
37	8 48 18.6200	9	.30756145	.66703065	.67858387	-.17201397	.71372314	.67897752	+ 8 820 250.4	13.7
	11 16 07.6970	10	-	-	-	-	-	-	+ 4 184 296.6	
		1	-.47575789	.86243359	.17280838	.63320833	.75458792	.17217519	+ 1 513 510.9	
38	8 48 26.6200	9	.30820331	.65886675	.68622542	-.17462300	.70573351	.68662000	+ 8 800 770.6	21.7
	11 16 15.7189	10	-	-	-	-	-	-	+ 4 230 179.9	

(continuation)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
		10	.06910734	.79138530	.60739895	.07712420	.79058266	.60747915	+ 1 250 061.6	
39	8 50 50.6200	1	-.46899152	.80736079	.35807192	.60879449	.70823735	.35744810	+ 8 407 889.4	23.9
	11 18 40.1132	9	.31552039	.51028684	.80003389	-.21616169	.55907987	.80043977	+ 5 036 718.1	
		10	.07136531	.78253173	.61850715	.07234779	.78232929	.61859041	+ 1 235 382.4	
40	8 50 58.6200	1	-.46827912	.80310799	.36841852	.60692106	.70453754	.36779568	+ 8 383 868.9	10.6
	11 18 48.1351	9	.31570834	.50209719	.80512524	-.21815455	.55093343	.80553146	+ 5 080 362.5	
		10	.07361943	.77352945	.62946992	.06855767	.77392429	.62955623	+ 1 220 632.6	
41	8 51 06.6200	1	-.46753918	.79872018	.37875213	.60500059	.70070803	.37813030	+ 8 359 474.4	28.8
	11 18 56.1570	9	.31589465	.49390986	.81010100	-.22013609	.54278687	.81050756	+ 5 123 822.7	
		10	.07583322	.76440908	.64025625	.06429551	.76539117	.64034556	+ 1 205 918.9	
42	8 51 14.6200	1	-.46675573	.79420467	.38907330	.60301894	.69675878	.38845254	+ 8 334 880.8	22.9
	11 19 04.1789	9	.31605471	.48573772	.81496520	-.22207979	.53464843	.81537207	+ 5 167 176.2	
		10	.07802893	.75517126	.65086701	.06003974	.75673459	.65095929	+ 1 191 216.5	
43	8 51 22.6200	1	-.46593662	.78956683	.39935862	.60098494	.69269354	.39873897	+ 8 310 121.1	20.4
	11 19 12.2008	9	.31619992	.47761187	.81969782	-.22399129	.52655078	.82010497	+ 5 210 340.6	
		10	.08020831	.74580950	.66131296	.05578771	.74794849	.66140820	+ 1 176 462.8	
44	8 51 30.6200	1	-.46507574	.78479176	.40964186	.59889017	.68849867	.40902340	+ 8 285 051.6	14.7
	11 19 20.2227	9	.31633640	.46948352	.82432792	-.22588542	.51844710	.82473535	+ 5 253 408.5	
		10	.08236483	.73633498	.67158532	.05154805	.73904269	.67178349	+ 1 161 706.9	
45	8 51 38.6200	1	-.46417507	.77988956	.41989733	.59673859	.68418364	.41928010	+ 8 259 763.7	19.7
	11 19 28.2446	9	-	-	-	-	-	-	+ 5 296 333.8	
		10	.08448261	.72675728	.68167922	.04733828	.73002382	.68178027	+ 1 147 022.4	
46	8 51 46.6200	1	-.46324776	.77485783	.43011260	.59454287	.67974365	.42949663	+ 8 234 272.6	34.8
	11 19 36.2665	9	.31655133	.45332219	.83324321	-.22957188	.50231721	.83365111	+ 5 339 094.3	
		10	.08658218	.71708032	.69159189	.04313910	.72089940	.69169579	+ 1 132 302.9	
47	8 51 54.6200	1	-.46227666	.76968922	.44031669	.59228510	.67517440	.43970205	+ 8 208 527.4	53.9
	11 19 44.2884	9	.31663544	.44527292	.83754046	-.23137274	.49427609	.83794856	+ 5 381 761.0	
		10	.08860327	.70731088	.70132030	.03895214	.71167610	.70142704	+ 1 117 551.6	
48	8 52 02.6200	1	-.46124646	.76441613	.45046609	.58995613	.67051040	.44985285	+ 8 182 590.7	13.7
	11 19 52.3103	9	.31670482	.43724700	.84173221	-.23314580	.48625344	.84214050	+ 5 424 252.3	
		10	.10974074	.59507770	.79614038	-.00543511	.60490910	.79627599	+ 955 802.0	
49	8 53 30.6270	1	-.44750129	.69796357	.55909700	.56088526	.61113233	.55850247	+ 7 881 780.2	10.7
	11 21 20.5582	9	.31661081	.35093917	.88124871	-.25086288	.39968382	.88165790	+ 5 882 029.4	
		10	.11149625	.58458497	.80363487	-.00929476	.59486363	.80377259	+ 941 098.3	
50	8 53 38.6270	1	-.44601292	.69120350	.56860372	.55792328	.60504949	.56801121	+ 7 853 059.1	19.2
	11 21 28.5802	9	.31654316	.34327900	.88428500	-.25232970	.39197692	.88469419	+ 5 922 684.3	
		10	.11321390	.57400552	.81094475	-.01311015	.58476116	.81108514	+ 926 404.2	
51	8 53 46.6270	1	-.44449769	.68432027	.57803768	.55492111	.59884660	.57744723	+ 7 824 120.9	45.3
	11 21 36.6021	9	.31645166	.33565760	.88723859	-.25375763	.38430364	.88764776	+ 5 963 210.6	
		10	.11489651	.56351457	.81807709	-.01688480	.57465750	.81821981	+ 911 841.3	
52	8 53 54.6270	1	-.44293686	.67733419	.59738857	.55186393	.59254678	.58680024	+ 7 795 004.5	37.8
	11 21 44.6240	9	.31635274	.32808117	.89010319	-.25516188	.37666992	.89051232	+ 6 003 599.5	

(continuation)

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
53	8 54 02.6270 11 21 52.6459	10 1 9	.11655904 -.44133764 .31626809	.55294275 .67022796 .32053693	.82502625 .59667040 .89287769	-.02063160 .54875612 -.25656608	.56450573 .58613165 .36906984	.82517127 .59608424 .89328679	+ 837 160.5 + 7 765 624.9 + 6 043 792.5	30.8
54	8 54 10.6270 11 22 00.6678	10 1 9	.11820379 -.43971237 .31614837	.54237030 .66300860 .31304122	.83178262 .60586517 .89557546	-.02434939 .54561118 -.25791887	.55434621 .57360596 .36151022	.83192992 .60528122 .89598450	+ 882 558.2 + 7 736 084.9 + 6 083 857.5	68.5
55	8 54 18.6270 11 22 08.6897	10 1 9	.11979189 -.43802238 .31602791	.53176447 .65569681 .30555607	.83837727 .61497812 .89819923	.02800528 .54239466 -.25926052	.54414020 .57299654 .35395945	.83852677 .61439646 .89860820	+ 867 936.5 + 7 706 296.2 + 6 123 838.1	43.1
56	8 54 26.6270 11 22 16.7116	10 1 9	.12137201 -.43631286 .31590595	.52117757 .64827475 .29811073	.84477380 .62399595 .90074049	-.03163849 .53914826 -.26058543	.53394749 .56627811 .34644627	.84492549 .62341661 .90114939	+ 853 312.2 + 7 676 264.2 + 6 163 519.8	30.6
57	8 55 30.6240 11 23 20.8838	1 10 -	-.42138336 .13296856 -	.58533707 .43662746 -	.69268794 .88976166 -	.51164910 -.05922842 -	.50909023 .45223604 -	.69212884 .88992952 -	+ 737 004.2 + 7 428 404.3 + 6 475 510.2	2.5
58	8 55 38.6240 11 23 28.9057	1 10 -	-.41936973 .13429482 -	.57705970 .42613242 -	.70080749 .89463739 -	.50803577 -.06249262 -	.50154563 .44205759 -	.70025112 .89480711 -	+ 722 523.5 + 7 396 440.6 + 6 513 752.1	3.5
59	8 55 46.6240 11 23 36.9276	1 10 -	-.41731780 .13558334 -	.56871728 .41567244 -	.70880640 .89935176 -	.50438466 -.06570080 -	.49393733 .43190423 -	.70825281 .89952329 -	+ 708 101.7 + 7 364 352.1 + 6 551 861.9	59.8
60	8 55 54.6240 11 23 44.9495	1 10 -	-.41524445 .13685883 -	.56028174 .40524038 -	.71669828 .90390812 -	.50070636 -.06888002 -	.48623646 .42177278 -	.71614750 .90408145 -	+ 693 660.5 + 7 331 973.0 + 6 589 731.9	39.0
61	8 56 02.6240 11 23 52.9714	1 10 -	-.41314747 .13810247 -	.55177842 .39484120 -	.72446514 .90831059 -	.49700291 -.07201093 -	.47846838 .41166518 -	.72391721 .90848567 -	+ 679 259.5 + 7 299 381.4 + 6 627 369.7	62.8
62	8 56 10.6240 11 24 00.9933	1 10 -	-.41101922 .13931892 -	.54318160 .38448376 -	.73213179 .91255820 -	.49326279 -.07509679 -	.47060883 .40159096 -	.73158673 .91273499 -	+ 664 894.2 + 7 266 606.3 + 6 664 964.4	27.4
63	8 56 18.6240 11 24 09.0152	1 10 -	-.40885575 .14051706 -	.53453043 .37415746 -	.73967168 .91665760 -	.48948844 -.07814809 -	.46269706 .39154117 -	.73912956 .91683608 -	+ 650 507.1 + 7 233 625.2 + 6 702 318.9	26.0
64	8 56 26.6240 11 24 17.0371	1 10 -	-.40666837 .14168671 -	.52580798 .36387346 -	.74709224 .92060903 -	.48568845 -.08115305 -	.45471446 .38152524 -	.74655307 .92078916 -	+ 636 158.0 + 7 200 440.2 + 6 739 507.6	46.7
65	8 56 34.6240 11 24 25.0590	1 10 -	-.40445685 .14283011 -	.51702566 .35363725 -	.75438659 .92441347 -	.48186451 -.08411309 -	.44667229 .37154902 -	.75385042 .92459521 -	+ 621 841.2 + 7 167 069.2 + 6 776 495.0	66.6
66	8 56 42.6240 11 24 33.0809	1 10 -	-.40221821 .14393890 -	.50818003 .34344019 -	.76155996 .92807889 -	.47801324 -.08702136 -	.43856758 .36160274 -	.76102682 .92826222 -	+ 607 575.8 + 7 133 506.6 + 6 813 371.2	37.0

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